Fibonacci Numbers Exercises

1. Recall the Sanskrit Poetry problem: How many poems of length n can be made with sequences of long syllables of length two and short syllables of length 1? Prove that the answer is the Fibonacci sequence. (Hint: Think about placing the first syllable and the resulting cases)

2. How does the previous problem relate to this one 'How many ways are there of covering a 2xn rectangle with 2x1 dominoes'? (Oxford Maths interview question)

3. How many binary sequences are there with no consecutive zeroes?

4. If Fibonacci took $F_0 = 2$ and $F_1 = 1$ so that his sequence was $2, 1, 3, 4, 7, 11, \ldots$, what is the limit

$$\lim_{n \to \infty} \frac{F_{n+1}}{F_n}$$

   Does this limit change at all based on our initial choice? ($F_0 = 2, F_1 = 1$ gives a sequence called Lucas Numbers)

5. What if instead we changed the recurrence relation e.g $F_0 = F_1 = 1$ and $F_n = F_{n-1} + 2F_{n-2}$ - what is the limit

$$\lim_{n \to \infty} \frac{F_{n+1}}{F_n}$$

6. Evaluate the limit

$$\lim_{n \to \infty} \frac{F_n}{2^n}$$